A Fundamental Theory of Sailing and its application to the design of a Hydrofoil Sail Craft

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Abstract

This paper presents a general comprehensive but succinct theoretical framework for analysing the forces acting on a sail craft and the resultant sail craft performance. An innovative type of hydrofoil sail craft, whose design was guided by the theory, is described. This type of craft should have superior performance to all existing types of high performance sail craft on all courses in most conditions. The theoretical analysis shows that the system of forces acting on any sail craft at equilibrium can be reduced to an equivalent system of three forces acting in a vertical plane. The resultant forces represent the net aero, hydro and gravitational forces. The geometrical relationships between these forces and the air/water/craft velocity triangle in the horizontal plane leads to a fundamental equation governing the limits of sail craft performance. Consideration is given to the implications of the theory regarding the necessary attributes of high performance sail craft in general. The particular type of hydrofoil sail craft described in the paper would be almost fully airborne when in use. A single inclined aerofoil and a single submerged inclinable hydrofoil would generate the main aerodynamic and hydro forces that would support and propel the craft.

List of symbols

AR	aspect ratio, i.e. ratio of span squared to area
atm.	unit of standard atmospheric pressure
c_d	profile or section drag coefficient
C	coefficient
CG	centre of gravity
CP	centre of pressure
D	drag, i.e. component of force parallel to flow
F	force
F_n	Froude number $F_n = V_S / \sqrt{g\ L_{WL}}$
g	acceleration due to gravity, $g \approx 9.8$ m/s^2
kgf	kilograms of force, 1 kgf≈9.8 newtons
L	lift, i.e. component of force perpendicular to flow
L_{WL}	waterline length
P	vertex of the velocity triangle
S	area of foil
V	velocity, speed
V_{mg}	velocity made good, i.e. component of sail craft speed parallel to the true wind
W	displacement/weight, $g\Delta$

α (alpha)	angle of attack or incidence
α_0	zero lift angle
β (beta)	apparent wind angle, i.e. angle between V_S and V_A or course with respect to apparent wind
γ (gamma)	course angle with respect to true wind
δ (delta)	sail trim angle
Δ (Delta)	displacement mass
ε (epsilon)	drag angle, i.e. arctan(D/L)
θ (theta)	elevation of CP_A with respect to CP_H
λ (lambda)	1. leeway angle 2. wavelength
ρ (rho)	fluid density, $\rho_A \approx 1.2$ kg/m^3, $\rho_H \approx 1000$ kg/m^3
ϕ (phi)	elevation, i.e. angle above the horizontal
ω (omega)	relative position of CG

Indices

A	air, aerodynamic, apparent wind
D	drag, i.e. component of force parallel to flow
H	water, hydrodynamic, hydrostatic
I	horizontal (mnemonic - air water interface) component, projection
L	lift, i.e. component of force perpendicular to flow
max	maximum
min	minimum
P	pressure
S	sail craft
T	true wind
V	vertical component

Mathematical signs and abbreviations

$\approx$	approximately equal to
$\ll$	much less than
$\mid$	such that, i.e. introduces condition

1 Introduction

This paper analyses the problem of the fundamental limitations on ultimate sail craft performance. The general attributes required for high performance sail craft become apparent from the analysis. Guided by the analysis a novel hydrofoil sail craft has been designed, and is described.

A few well-understood basic principles are applied to the problem. The primary principles come from simple Euclidean geometry and statics, that branch of engineering mechanics that deals with rigid bodies at equilibrium under a system of forces. Results from aerofoil theory, which is equally applicable to foils operating submerged in water, are also used. Basic results from the hydrodynamic and hydrostatic theory applicable to the motion of a hull on the water surface are also needed. The principal initially used from aerofoil theory is the simple fact that lift, a force component perpendicular to the direction of flow, can be generated.

An innovative approach is taken to the analysis of the complete system of forces acting on a sail craft. The system of forces acting on a sail craft at equilibrium is reduced to an equivalent system of three resultant forces with concurrent lines of action lying in a vertical plane. The resultant forces represent the net aero, hydro and gravitational forces. The geometrical relationships between these forces and the air/water/craft velocity triangle in the horizontal plane leads to a fundamental equation governing the limits of sail craft performance. The equation relates the apparent wind angle to basic parameters associated with the net aero and hydro forces.

The approach taken in reducing the system of forces obviates the need to give independent consideration to heeling, pitching and righting moments.

The theory is applicable to all sail craft. This includes displacement yachts, dinghies, multihulls, sailboards and kite powered craft. The theory can be applied to ice and land yachts, with appropriate substitutions for references to water. While the theory may apply to all sail craft, the emphasis is on high performance, meaning high speed relative to the true wind speed, and high absolute speed.

The hydrofoil sail craft described in the paper would be almost fully airborne when in use. A single inclined aerofoil and a single submerged inclinable hydrofoil would generate the main aerodynamic and hydro forces that would support and propel the craft. The craft should have superior performance to all existing types of high performance sail craft on all courses in most conditions.

Section outline

The paper is structured as follows. Section 2 introduces the velocity triangle and the arc of constant apparent wind angle. The dependence of various performance measures on apparent wind angle is presented. An expression is given for a velocity ratio that changes and must be accommodated as course changes.

In Section 3 the horizontal components of the net aerodynamic and hydrodynamic forces are examined and their geometric relationship to the velocity triangle.

In the next section the full three-dimensional system of forces is considered. These are reduced to three resultant forces, representing the net aero, hydro and gravitational forces. The aero and hydro force to weight ratios are expressed as functions of the force elevation angles. An equation relating the apparent wind angle to the drag angles and the force elevation angles is derived. This is the fundamental equation determining sail craft performance. The relatively low hull drag at low Froude number is discussed. The basic equations for fluid dynamic lift and drag are introduced and discussed in relation to the velocity ratio variation that accompanies changes in course. Finally in this section the relevant literature is reviewed.

In Section 5 the general principles are considered regarding the relative locations of the centre of gravity and the aerodynamic and hydro centres of pressure, and the elevation angles of the resultant force lines of action. The attributes required for high performance and for stable equilibrium are deduced. The design implications of aspect ratio and induced drag are discussed. Finally the designs of a variety of existing types of sail craft are reviewed.

Finally in Section 6 the novel hydrofoil sail craft design is presented. Cavitation is discussed. Alignment of the craft in response to the course, apparent wind and net forces is shown. The resultant forces and lines of action in the vertical plane are shown. The craft geometry is parameterised and an equation derived relating the resultant force elevation angles. Representative values are assumed for the craft geometry and drag angles, and a full analysis of relative craft performance is undertaken. Finally specific performance measures are derived for a particular example craft weight, aerofoil area and true wind.

2 The velocity triangle and the arc of constant apparent wind angle β

The velocity triangle showing the relationship between the true wind V_T, the sail craft speed V_S and the apparent wind V_A is depicted in Figure 1. The course γ with respect to the true wind and the apparent wind angle β are also shown in the figure. The sail craft speed made good V_{mg} is shown, and is defined to be positive in the upwind direction and negative in the downwind direction.

The following property comes from Euclidean geometry.

> *The angle subtended at any point on a circular arc by its chord is constant, and equal to the angle between the chord and the endpoint tangents.*

The property can be applied to the velocity triangle. For V_T and β fixed, the locus of the vertex P is a circular arc as shown in Figure 1. For the example portrayed in the figure $\beta<\pi/2$, and this must be true for all craft sailing to windward or even on a beam reach, and for all craft sailing faster than the wind, including craft sailing downwind faster than the wind. In other cases it need not be true. The focus of this paper is on high performance craft that can achieve $\beta<<\pi/2$ over a wide range of courses. Much of the material presented is equally applicable for $\beta\geq\pi/2$, and it is generally clear or easily verifiable when this is the case.

The best speed made good upwind $V_{mg_{\max}}$ is achieved on the course angle $\gamma|_{V_{mg_{\max}}}$ that corresponds to P_1 in Figure 1. The best speed made good downwind $V_{mg_{\min}}$ is achieved on the course corresponding to P_5. Courses of practicable interest range between these two extremes. The maximum craft speed $V_{S_{\max}}$ occurs when the vector V_S passes through the centre of the circle and ends at P_4. The remaining points P_2 and P_3 correspond to $V_{A_{\max}}$ and $V_A=V_S$ respectively. Expressions for various parameters are given for each of the five points in Table 1. Some parameters of particular interest are plotted as functions of β in Figure 2. Small β is required for high performance.

Velocity ratios on differing courses

As the course γ changes the relativity between V_A and V_S changes. At the extremes

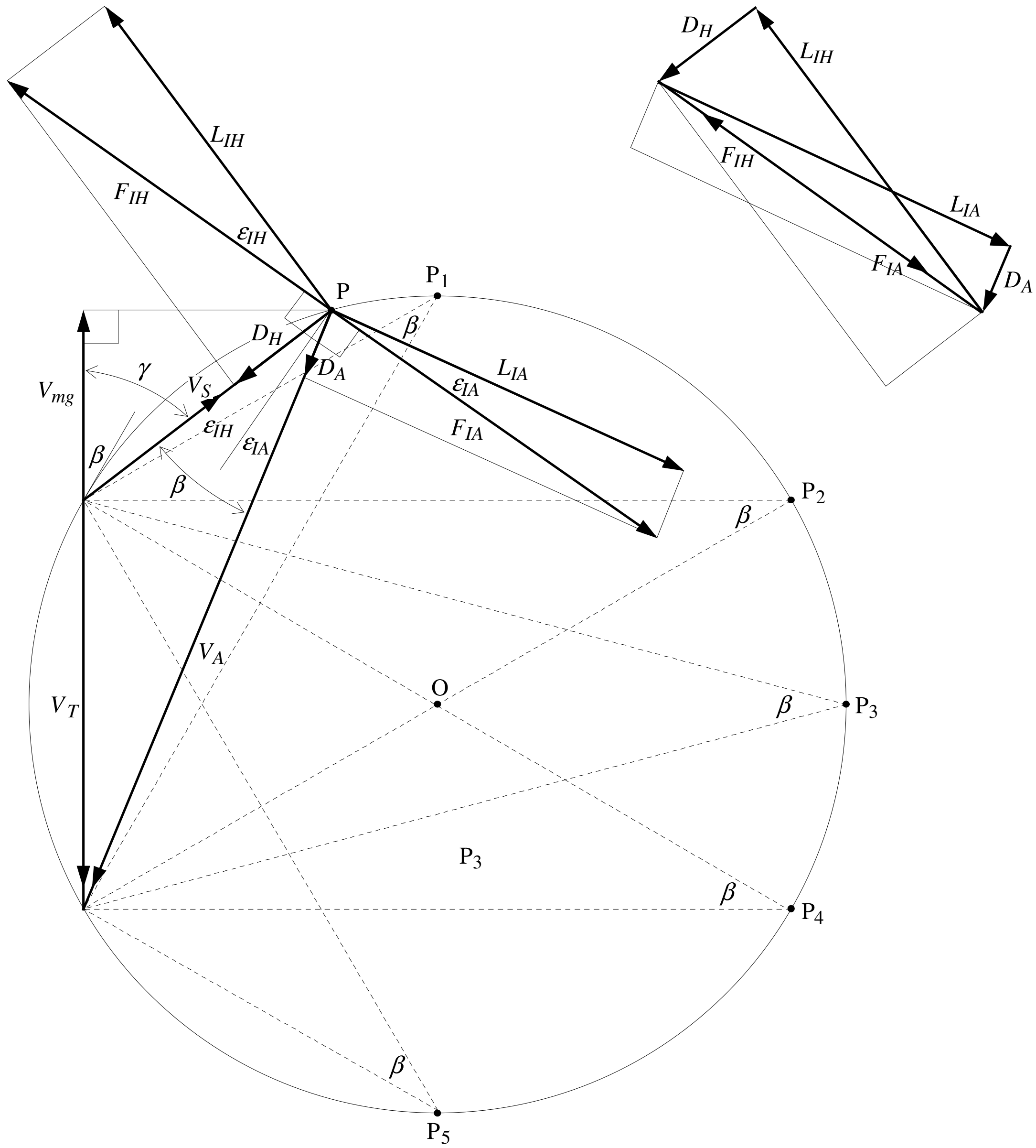

Figure 1 *Velocities, forces and angles in the horizontal plane.*

$$\left.\frac{V_A}{V_S}\right|_{V_{mg\,\max}} = \left.\frac{V_S}{V_A}\right|_{V_{mg\,\min}} = \sqrt{\frac{1+\sin\beta}{1-\sin\beta}}$$

and the square of these ratios

$$\left.\frac{V_A{}^2}{V_S{}^2}\right|_{V_{mg\,\max}} = \left.\frac{V_S{}^2}{V_A{}^2}\right|_{V_{mg\,\min}} = \frac{1+\sin\beta}{1-\sin\beta}. \tag{1}$$

	Course angle γ	Boat speed V_S/V_T	Apparent wind speed V_A/V_T	Speed ratio V_A/V_S	Boat speed made good V_{mg}/V_T	Remarks
P_1	$\frac{\pi}{4}+\frac{\beta}{2}$	$\frac{\sin(\pi/4-\beta/2)}{\sin\beta}$	$\frac{\sin(\pi/4+\beta/2)}{\sin\beta}$	$\sqrt{\frac{1+\sin\beta}{1-\sin\beta}}$	$\frac{1}{2}\frac{1}{\sin\beta}-\frac{1}{2}$	$V_{mg_{\max}}$
P_2	$\frac{\pi}{2}$	$\frac{1}{\tan\beta}$	$\frac{1}{\sin\beta}$	$\frac{1}{\cos\beta}$	0	$V_{A_{\max}}$
P_3	$\frac{\pi}{2}+\frac{\beta}{2}$	$\frac{1}{2}\frac{1}{\sin(\beta/2)}$	$\frac{1}{2}\frac{1}{\sin(\beta/2)}$	1	$-\frac{1}{2}$	$V_S=V_A$
P_4	$\frac{\pi}{2}+\beta$	$\frac{1}{\sin\beta}$	$\frac{1}{\tan\beta}$	$\cos\beta$	-1	$V_{S_{\max}}$
P_5	$\frac{3\pi}{4}+\frac{\beta}{2}$	$\frac{\sin(\pi/4+\beta/2)}{\sin\beta}$	$\frac{\sin(\pi/4-\beta/2)}{\sin\beta}$	$\sqrt{\frac{1-\sin\beta}{1+\sin\beta}}$	$-\left(\frac{1}{2}\frac{1}{\sin\beta}+\frac{1}{2}\right)$	$V_{mg_{\min}}$

Table 1 *Relative speeds for selected course angles.*

These values are both plotted in Figure 2. Small values are beneficial. The reason will be explained in Section 4. Small values occur when β is small.

3 Horizontal components of the net aerodynamic and hydrodynamic forces

The net aerodynamic and hydrodynamic horizontal force components F_{IA} and F_{IH} are represented in Figure 1. The indices A and H signify aero and hydro respectively, while I signifies horizontal. A mnemonic for I is the *interface* between air and water. The force components F_{IA} and F_{IH} can be further separated into the horizontal components of lift L_{IA} and L_{IH} and total drag D_A and D_H. The angles between the horizontal components of force and lift are denoted ε_{IA} and ε_{IH}.

For equilibrium F_{IA} and F_{IH} must share a common line of action and be equal in magnitude but opposite in direction. The vector sums are represented pictorially in the inset to Figure 1.

From the superposition of the force components on the velocity triangle as in Figure 1 it can readily be seen that

$$\beta = \varepsilon_{IA} + \varepsilon_{IH} \, . \tag{2}$$

Small ε_{IA} and ε_{IH} are required for high performance. Note that ε_{IA} and ε_{IH} should not be referred to as the aerodynamic and hydrodynamic drag angles. In accordance with the conventional nomenclature used in fluid dynamics these titles are reserved for related angles to be introduced in the next section. Figure 3 represents a conventional

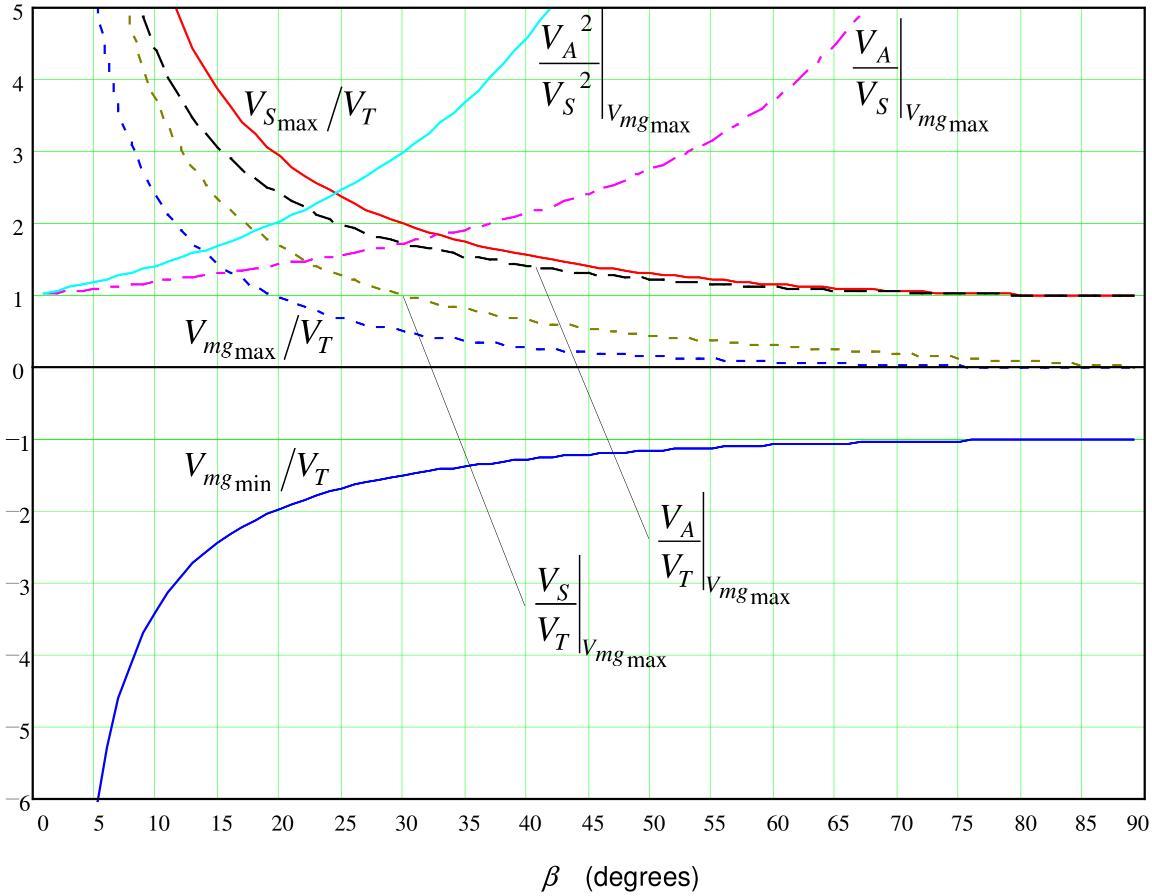

Figure 2 *Relative speeds and speed ratios as functions of β.*

sail craft with a generally vertical sail and fin keel or centreboard. It shows the leeway angle λ, which is identical to the fin or centreboard angle of attack or incidence α_H, the sail trim angle δ and the sail incidence angle α_A. Notice that

$$\beta = \alpha_A + \delta + \alpha_H \,.$$

It is important to distinguish α_A from ε_{IA} and α_H from ε_{IH}.

Figure 3 also introduces a vector representing the apparent water flow V_H with respect to the reference frame of the craft. The vector V_H is the reverse of V_S.

4 The net aero, hydro and gravitational forces

Analysis of the totality of forces acting on a sail craft is assisted by the following results from the sub branch of engineering science known as statics. Statics, in contrast to dynamics, deals with the mechanics of rigid bodies at equilibrium under a general force system.

> *Any system of forces can be reduced to an equivalent single force vector and a collinear couple vector.*

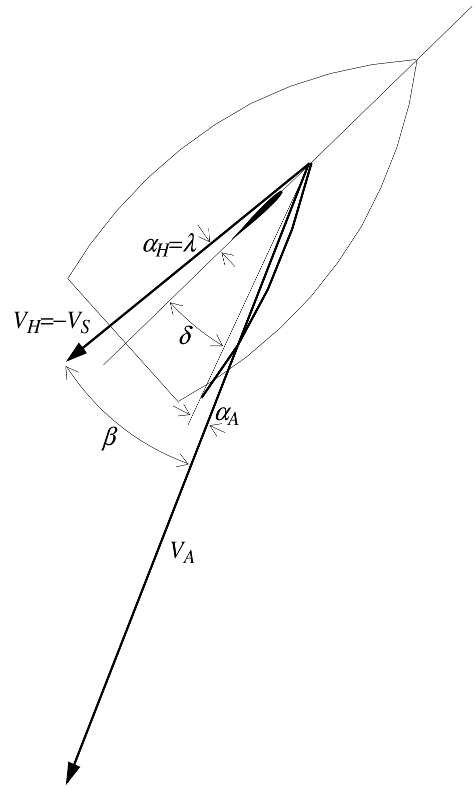

Figure 3 *Leeway, sail trim and incidence angles for conventional sail craft.*

Couple is synonymous with moment or torque. This force-couple system is called a wrench. It can be thought of as a push and twist along a single axis. A reference is [1].

> *If three forces, acting on a rigid body, produce equilibrium, their directions must lie in one plane; and must all meet in one point, or be parallel* [2].

Equilibrium exists when the vector sum of the forces is zero, and there is no net moment. More succinctly, the lines of action must be coplanar; and must be concurrent or parallel.

All of the forces acting on a sail craft belong to three distinct classes: aero, hydro and gravitational. The resultant of the gravitational forces is a single force, the weight W, acting vertically downwards through the centre of gravity CG. The resultant of the aerodynamic forces is a single force F_A acting through the aerodynamic centre of pressure CP_A, and possibly a residual moment. For most craft it is reasonable to assume that the residual moment, if it exists at all, is negligible. The hydro forces include hydrodynamic forces acting on hulls and foils, and hydrostatic buoyancy. A major component of drag may be wave-making resistance. The resultant of the hydro forces is a single force F_H acting through the hydro centre of pressure CP_H, and possibly a residual moment, which similarly will be assumed to be negligible or non-existent.

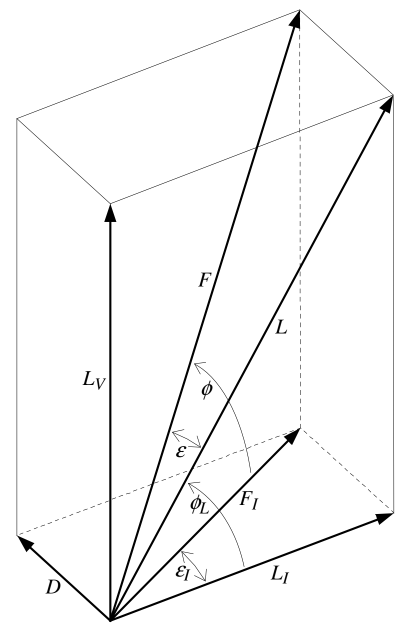

Figure 4 *Net force, force components, and projection onto the horizontal plane.*

For either fluid, air or water, the force F may be separated into the components lift L and drag D, as shown in Figure 4, which are normal and parallel to the direction of flow, respectively. The lift L can be further separated into vertical lift L_V and horizontal lift L_I. The horizontal force component F_I is also shown in the figure. The angle ε between force F and lift L will be referred to as the aerodynamic or hydrodynamic drag angle, as appropriate, notwithstanding that in the hydro case there may be a significant hydrostatic contribution to the vertical lift L_V. The angle ε_I between the horizontal force and lift components F_I and L_I is the projection of the drag angle ε onto the horizontal. The angle ϕ is the elevation of force F with respect to the horizontal. In the figure ϕ is depicted as positive, but it could be zero or negative. The angle ϕ_L is the elevation of lift L with respect to the horizontal. This too may assume positive, zero or negative values. If a single foil generated the force F, then the angle ϕ_L would be the roll angle of the foil's lateral axis measured from the vertical. The angular relationships may be expressed algebraically by

$$\sin\varepsilon = \sin\varepsilon_I \cos\phi, \quad \text{and}$$

$$\sin\phi = \sin\phi_L \cos\varepsilon.$$

Add the indices A or H to all of the above values to denote the specific aero or hydro quantity. The angular relationships are

$$\sin\varepsilon_A = \sin\varepsilon_{IA} \cos\phi_A, \tag{3}$$

$$\sin\varepsilon_H = \sin\varepsilon_{IH} \cos\phi_H, \tag{4}$$

$$\sin\phi_A = \sin\phi_{LA} \cos\varepsilon_A, \quad \text{and} \tag{5}$$

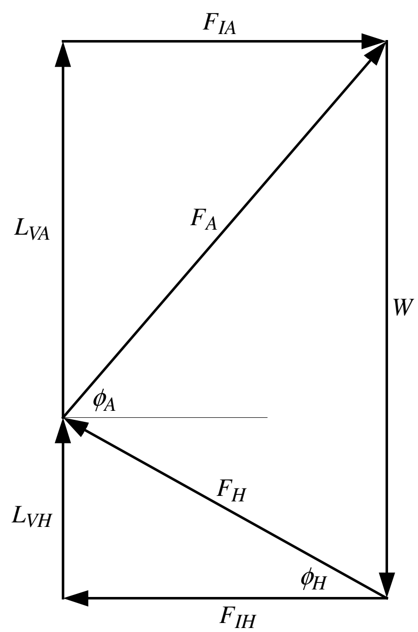

Figure 5 *Vector sum of net forces.*

$$\sin\phi_H = \sin\phi_{LH} \cos\varepsilon_H . \qquad (6)$$

To picture the totality of forces acting on a sail craft visualise aero and hydro versions of Figure 4 built up on the foundations in Figure 1, together with the weight W acting vertically downwards through P. Note that

$$L_{VA} + L_{VH} = W .$$

The vector sum of the resultant forces in the vertical plane in which they act is represented in Figure 5. The relationships can be expressed algebraically by

$$\frac{F_A}{W} = \frac{\cos\phi_H}{\sin(\phi_A + \phi_H)}, \quad \text{and} \qquad (7)$$

$$\frac{F_H}{W} = \frac{\cos\phi_A}{\sin(\phi_A + \phi_H)} . \qquad (8)$$

The fundamental equation

Using (3) and (4) to substitute for ε_{IA} and ε_{IH} in equation (2) yields

$$\beta = \arcsin\left(\frac{\sin\varepsilon_A}{\cos\phi_A}\right) + \arcsin\left(\frac{\sin\varepsilon_H}{\cos\phi_H}\right), \qquad (9)$$

which relates the apparent wind angle β to the drag angles ε_A and ε_H, and to the elevation angles ϕ_A and ϕ_H. This is the fundamental equation governing sail craft performance. High performance requires small β. From the equation it is clear that this requires small drag angles and small elevation angles, but with rapidly diminishing returns as ϕ_A and ϕ_H tend to zero. Small elevation angles result from large aerodynamic and hydro force to weight ratios F_A/W and F_H/W. More succinctly, the equation gives analytical rigour to the intuitive notion that high performance requires a high force to weight ratio and low drag. The equation also provides the major reason why sail craft apparent wind angles are so much greater than the glide angles achieved by sailplanes, a question raised by Bethwaite [3].

The theory presented above applies to all sail craft. The theory is equally applicable to land and ice yachts if references to hydro forces are replaced with the forces generated by the wheels or runners respectively. For conventional craft the sail is generally vertical, and so $\phi_A \approx 0$, however ϕ_H is generally much greater than zero. The Froude number is the non-dimensional value

$$F_n = \frac{V_S}{\sqrt{g\ L_{WL}}}$$

where g is the acceleration due to gravity and L_{WL} is the waterline length of the craft. The speed of deep-water waves of wavelength λ is

$$\sqrt{\frac{g\ \lambda}{2\ \pi}}\ .$$

When $F_n = 1/\sqrt{2\pi}$ the boat speed corresponds to the speed of waves with wavelength $\lambda = L_{WL}$. For a conventional displacement craft the drag increases dramatically as F_n approaches $1/\sqrt{2\pi}$. At lower craft speed drag is very low and ε_H is typically very small, whereas ϕ_H is very large, resulting in a moderate value for ε_{IH}. Kite powered craft and sailboards generally have inclined aerofoils, which provide some of the vertical lift, so $\phi_A > 0$. On a conventional craft which is heeled the sail generates some negative vertical lift, and so $\phi_A < 0$, which is counter productive.

Accommodating velocity ratio variation

The lift L and drag D components of the force generated by a foil may be expressed as functions of the fluid density ρ, velocity V, foil area S, and the coefficients of lift C_L and drag C_D by

$$L = \frac{1}{2}\rho\, V^2 S\, C_L\,, \quad \text{and} \tag{10}$$

$$D = \frac{1}{2}\rho\, V^2 S\, C_D\,.$$

Over some useful working range of foil incidence angle α the coefficient of lift varies approximately linearly with respect to α

$$C_L \approx 2\pi(\alpha - \alpha_0)\,,$$

where α_0 is the incidence angle at which C_L=0. The coefficient of drag is a more complex function of α, and also depends on aspect ratio AR, section profile and Reynolds number Re. It suffices to say that C_D is much less than C_L over the useful working range of α. Incidentally, the presence of the fluid density ρ in the above equations explains why the aerofoils are so much larger than the hydrofoils. Recall from Section 2 that as the course angle γ varies so too does the relative speed of the air and water flow over the respective foils. In order to maintain equilibrium it will be necessary to vary one of the foil areas S_A or S_H, or one of the coefficients of lift C_{LA} or C_{LH}, or a combination of these, to compensate for the relative change in $V_A^{\ 2}$ and $V_H^{\ 2}$. The expression (1) gives a guide to the relative magnitude of the compensation required at the course extremes. Small values of β have the advantage of reducing the variation required to foil areas or coefficients of lift as the course γ changes.

The performance polar of a craft for which the apparent wind angle β remained constant over all courses γ would be a circular arc as shown in Figure 1. However for real craft the performance polar will not be perfectly circular due to changes in the drag angles ε_A and ε_H, and the elevation angles ϕ_A and ϕ_H, as adjustments occur to compensate for the changes in V_S and V_A. The most dramatic change is the wave making drag increase for a displacement hull as F_n approaches $1/\sqrt{2\pi}$. Detailed analysis of the forces generated by displacement hulls and planing hulls is given in [4].

Review of the literature

The full coherent method presented in this paper for the analysis of the total forces acting on a sail craft and the resultant speed performance is novel. However many individual parts of the complete theory have been appreciated in isolation for many years. Lanchester [5] in 1907 pointed out that

> *"the minimum angle at which the boat can shape its course relatively to the wind is the sum of the under and above water gliding angles."*

This is reproduced in Marchaj [6] along with its algebraic representation equivalent to equation (2), however in this paper the distinction is made that the angles to be summed are not referred to as the drag angles, but rather the projections onto the horizontal of the drag angles. Barkla [7] in 1971 pointed out that the polar diagram of ice yacht speed would be a circular arc, similar to that shown in Figure 1, but consideration was restricted to craft with zero runner friction and so $\varepsilon_H=0$ and $\beta=\varepsilon_A$. This too is reproduced in Marchaj [6]. Marchaj presents expressions similar to those appearing in Table 1 for $V_{mg_{\max}}$, $V_{S_{\max}}$ and $V_{mg_{\min}}$, and the corresponding course angles, but again restricted to the context of ice yachts with zero runner friction, and so the independent variable is ε_A rather than β. Bethwaite [3] in 1993, apparently independently of the ice yacht examples, gives examples of polar diagrams showing circular arcs for constant apparent wind β. However Bethwaite does not mention the decomposition of β into aerodynamic and hydrodynamic contributions as given by equation (2). Perry [8] in a 1998 report describing a hydrofoil sail craft design refers to

> *"concurrent lines of action for the gravity force, sail force and the windward hydrofoil force."*

5 Location and direction of the net forces

At the level of abstraction required by the above theory, the description of a sail craft comprises the relative locations of the centre of gravity CG, the aerodynamic centre of pressure CP_A and the hydro centre of pressure CP_H, and the elevations of the aerodynamic lift ϕ_{LA} and the hydro lift ϕ_{LH}. The locations and elevations may be variable so that feasible solutions can be obtained which satisfy the conditions for equilibrium over a range of V_T and γ. The hull and foils must be capable of generating the required forces.

The weight W acts vertically downwards so the requirement for concurrent lines of action can be simplified. The aerodynamic and hydro force lines of action must intersect on the vertical line through the centre of gravity CG. Clearly there must be some vertical component of separation between CP_A and CP_H. It follows that at least one of CG, CP_A and CP_H must be horizontally separated from the rest. Recall that high performance requires small elevation angles. The exceptions are ice and land yachts, and displacement watercraft with small Froude number, all of which incur extremely small drag penalties for vertical lift. Small elevation angles require that the horizontal separation be large relative to the unavoidable vertical separation.

The aspect ratio of a foil AR is defined to be the ratio of the span squared to the area, which for a rectangular foil reduces to the span to chord ratio. The coefficient of drag for a foil may be decomposed as follows

$$C_D = c_d + \frac{{C_L}^2}{\pi \, AR},$$

where c_d is the profile or section drag coefficient and varies with α. The second term is the induced drag. Low induced drag calls for high aspect ratio, and therefore large vertical separation between CP_A and CP_H. The largest contributor is of course the sail or aerofoil span. The inescapable conclusion is that while the virtue of a ship being tall is defended, it should also be relatively wide. An Orwellian motto for high speed, high performance sailing could be “tall ships good, wide ships better”.

It is generally preferable that neither elevation ϕ_A nor ϕ_H should be negative, in fact the sum of the projected drag angles would generally be less if ϕ_A and ϕ_H were similar. If multiple independent aerofoils were used, then parallel lines of action would be more efficient. The same principle holds for a combination of hydrofoils and planing surfaces. Note that this principle is not satisfied by a typical hull and vertical foil combination.

Stability of the equilibrium

Stability of the equilibrium is assisted if CP_A is to leeward of CP_H. In this sense stability refers to the general orientation of the craft, not its resistance to heeling. For example consider the instability of a long rod when two compressive forces in equilibrium are applied at opposite ends. The slightest misalignment would be immediately amplified. Compare this with the stability when two tensile forces are applied at opposite ends. Stability is also assisted if CG lies below the line from CP_A to CP_H. Natural stability reduces the need for the addition of stabilising features to the craft with their inevitable associated drag. If a foil is vertical, then the line of action is horizontal. It follows that the intersection of the line of action with the vertical line through CG is unaffected by the lateral position of CP_A. However stability of the equilibrium is affected by the lateral position of CP_A.

The combination of the requirements that the elevations ϕ_A and ϕ_H are non negative, CP_A is to leeward of CP_H, and CG lies below the CP_A to CP_H line, can only be satisfied if CG lies laterally between CP_A and CP_H.

Review of sail craft types

Some example classes of craft will now be considered. On a winged skiff CP_A remains vertically above CP_H and CG moves a variable amount to windward. For a large multihull CP_A remains vertically above CG and CP_H moves a variable amount to leeward. On a sailboard CG and CP_A move slightly to windward of CP_H, and $\phi_A>0$, but the interconnection of components prevents the achievement of small ϕ_H. For kite boards the kite may be a long way to leeward, but the kite and in particular the lines attaching it to the sailor, do not constitute a rigid body. Analysis could be conducted by considering the force exerted by the kite lines at their point of attachment to the board rider. An advantage is that a relatively high aspect ratio aerofoil is generating a force at the attachment point just a few feet above the water surface, however small ϕ_H cannot be achieved.

6 A high performance hydrofoil sail craft

The author has designed a novel type of hydrofoil sail craft, which should be capable of sailing at very high speeds and small apparent wind angles. It is fully described in the patent application [9]. A perspective drawing taken from that publication is reproduced at Figure 6. The numbered parts are described in [9].

The design goal was a craft of minimum necessary complexity that was capable of high performance over a wide range of true wind speed V_T and course γ. This capability was required on both tacks and in unsheltered waters.

All of the general principles for high performance and stability of equilibrium were considered and generally accommodated. The design choice process led to selection of the following general features. The locations for CG, CP_A and CP_H are fixed. In use a single aerofoil and a single submerged hydrofoil generate the main aerodynamic and hydro forces, respectively. Only the hydrofoil, together with its supporting struts and stabilisers, remains submerged, the rest of the craft being airborne. The main aerofoil and hydrofoil have fixed areas S_A and S_H.

The craft must be able to accommodate a range of projected drag angles ε_{IA} and ε_{IH}, and it must be capable of providing a range of force elevation angles ϕ_A and ϕ_H, and coefficients of lift C_{LA} and C_{LH}. This can be achieved as shown in Figure 6 by providing three degrees of freedom in the gimbal assemblies supporting the aerofoil and hydrofoil. Yaw rotation about the vertical axes maintains alignment with the air and water flow respectively, keeping the foil lateral axes transverse to the respective flows. Roll rotation about the horizontal axes controls the lift elevation angles ϕ_{LA} and ϕ_{LH}. Finally pitch rotation about the foil lateral axes changes the incidence angles α_A and α_H, thereby controlling the coefficients of lift C_{LA} and C_{LH}.

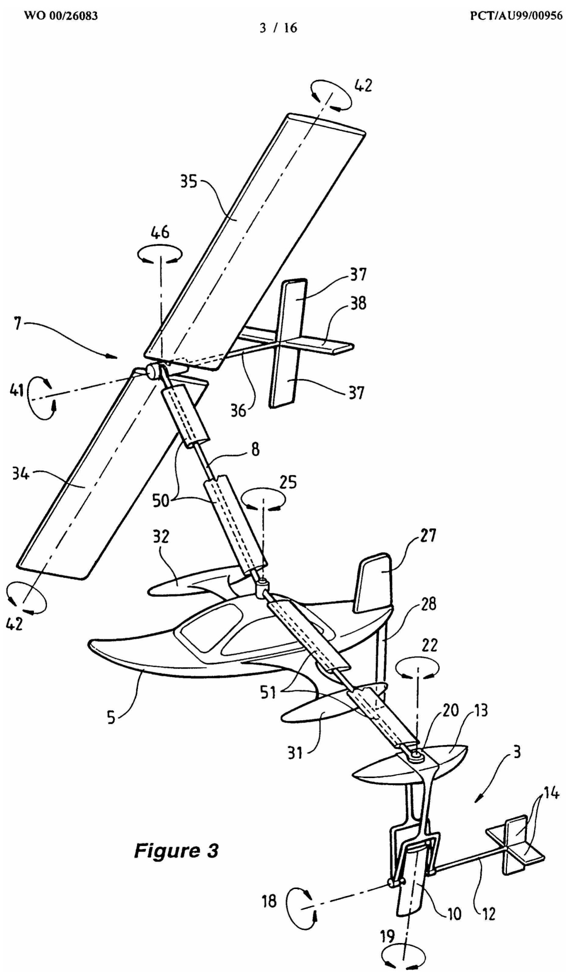

Figure 6 *Perspective drawing of a hydrofoil sail craft* [9].

In addition the hull is free to rotate, about a vertical axis, with respect to the main beam connecting the aerofoil and hydrofoil assemblies. Yaw rotation combined with rolling ϕ_L through $\pi/2$, that is rolling the foil to be horizontal and then beyond on the other side, allows the craft to sail on either tack. Furthermore the foils can be thick and asymmetric. This is necessary to provide a wide range in the coefficient of lift while maintaining low drag.

Since the ratio of foil areas S_A to S_H is fixed, it is desirable that the F_A to F_H ratio remains reasonably constant as the force magnitudes change. This is achieved if CG is close to midway laterally between CP_A and CP_H. Fortunately this is not inconsistent with other requirements.

Recall that each foil has three degrees of freedom associated with it. Yaw can be controlled automatically by provision of fins to maintain alignment with the fluid flow. The pilot must control roll and pitch. This could be achieved by provision of a joystick for each foil. Steering is achieved by generating transient net lateral forces by coordinated adjustment of the aerofoil and hydrofoil. The foil assemblies should be reasonably well balanced about their axes, both with respect to fluid dynamic pressure and inertial mass. The wind velocity gradient must be accommodated to fully achieve balance.

Further discussion and detail regarding the design and variations may be found in the patent application. The topics discussed include control and operation of the craft, including take off from rest and changing tack, stability, the use of stabilisers and elevators, and choice of foil section shapes and properties.

Use of a single submerged hydrofoil is virtually mandated by the stated design goals and the principles governing high performance. There are additional benefits. Hydrofoils have superior lift to drag ratios compared with planing surfaces. A submerged hydrofoil avoids the problems associated with the rough state of the water surface. It also avoids ventilation that can affect surface piercing foils. A disadvantage is the drag associated with the supporting struts. Ventilation may further increase drag on the struts.

Cavitation

The pressure distribution over a foil may be expressed by

$$\frac{1}{2}\rho V^2 C_P$$

where C_P is the coefficient of pressure and varies over the surface. The pressure distribution changes as the incidence angle α changes. When the magnitude of the pressure drop at some point on a hydrofoil surpasses the ambient fluid pressure, cavitation occurs, and performance is impaired. For example water at V_H=28 kn has dynamic pressure

$$\frac{1}{2}\rho_H \, V_H{}^2 \approx 1\,\mathrm{atm}\,.$$

Cavitation could be a performance limitation, and foil shape for high speed should be carefully selected to control C_{PH}. This requirement limits the foil thickness and reduces the maximum value and range of C_{LH}. Although control of C_{LH} is limited, equilibrium can still be achieved by adjusting C_{LA}, ϕ_{LA} and ϕ_{LH}. Ultimately further increases in speed will require the adoption of super cavitating hydrofoils, but these have markedly inferior lift to drag ratios.

Force geometry

Figure 7 is a plan view of a hydrofoil sail craft. Superimposed on it are vectors representing the water flow and apparent wind. Also superimposed are the horizontal force components. For simplicity the forces are shown acting at the aerofoil and hydrofoil gimbal centres. This is equivalent to assuming negligible drag on the main connecting beam and hull. Finite drag would slightly offset the craft total aerodynamic centre of pressure. The inset diagram shows the velocity triangle. The velocities and forces shown in Figure 7 are identical to those in Figure 1. Notice that the main connecting beam is aligned with the forces and lies in the resultant force plane, under the assumptions of negligible beam and hull drag. A bonus of this design layout is that the main structural component, the beam, tends to be in tension, rather than compression. This allows it to be a comparatively lighter structure, although there may be bending moments that it must withstand.

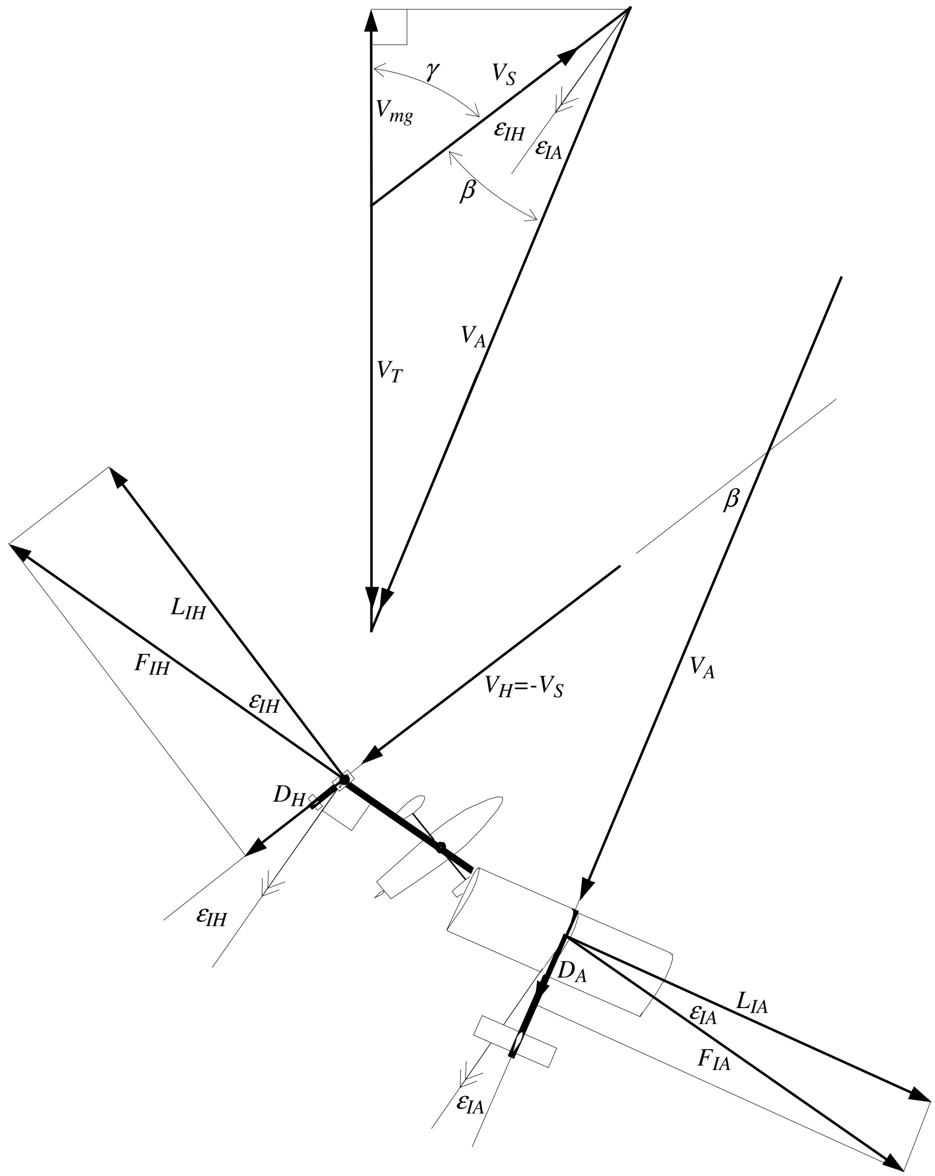

Figure 7 *Plan view of craft with forces and velocities overlaid.*

Figure 8 shows an offset frontal view of a hydrofoil sail craft. In fact it is a view normal to the resultant force plane. The waterline is represented in the figure, and all but the lower part of the hydrofoil assembly is airborne. The resultant forces and their lines of action are superimposed. The resultant forces and their lines of action are reproduced without the craft but with the force components shown, and the elevation angles ϕ_A and ϕ_H marked. To complete the force analysis the craft must be characterised by two further parameters. These are the elevation θ of the aerodynamic centre of pressure CP_A with respect to the hydrodynamic centre of pressure CP_H, and the lateral position of the centre of gravity CG expressed as a proportion ω of the lateral distance from CP_H to CP_A. These parameters are shown in the figure. Also

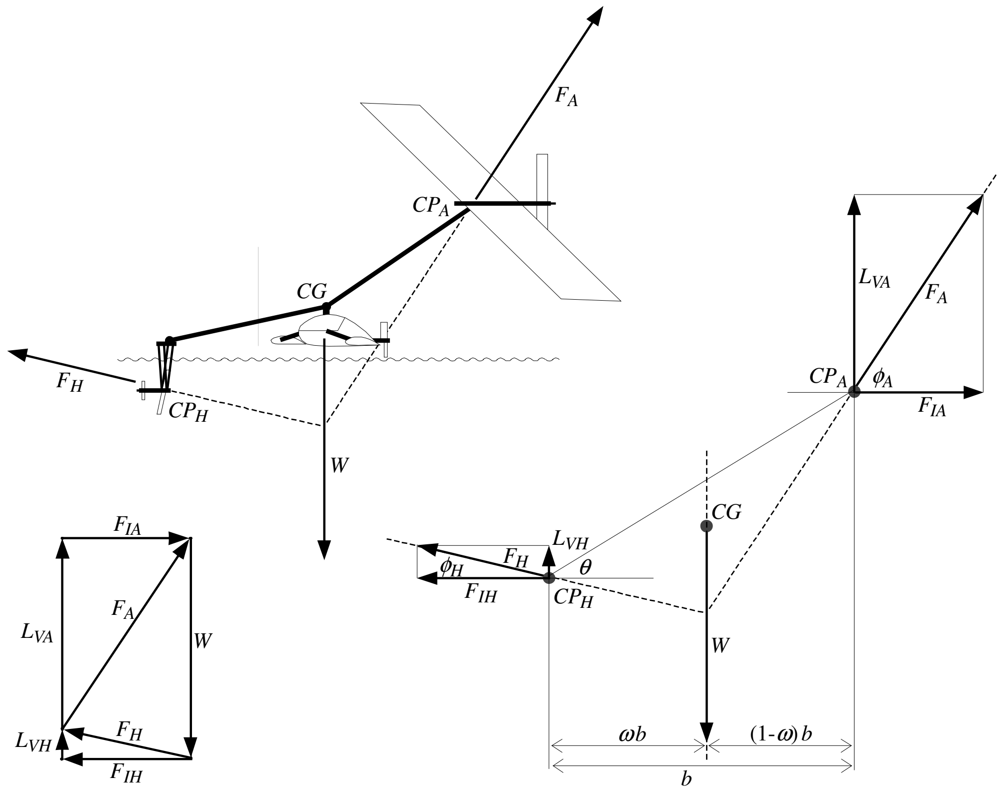

Figure 8 *Net forces in their vertical plane.*

shown in Figure 8 is a pictorial representation of the vector sums of the forces and their components.

The condition of concurrency of the force lines of action is equivalent to the algebraic constraint

$$(1-\omega)\tan\phi_A = \omega\tan\phi_H + \tan\theta\,. \tag{11}$$

This determines the relationship between the elevation angles ϕ_A and ϕ_H. The ranges of ϕ_A and ϕ_H are limited by

$$\theta \le \phi_A \le \frac{\pi}{2} - \varepsilon_A\,, \quad \text{and} \tag{12}$$

$$-\theta \le \phi_H \le \frac{\pi}{2} - \varepsilon_H\,.$$

The elevations ϕ_A and ϕ_H are related by (11), and so the upper bounds are determined by whichever is the most restrictive of these two conditions. Recall from the previous section that high performance requires a craft to be relatively wide. For the hydrofoil craft described in this section that is equivalent to requiring small θ. This is confirmed by (12), which states that ϕ_A is bounded below by θ.

Performance analysis

A hypothetical example will now be given to predict the performance that could reasonably be expected in practice. Modest parameter values that should be achievable are assumed. Let θ=30°, ω=0.45 and $\varepsilon_A = \varepsilon_H = 7.5°$. As shown in Figure 9, ϕ_A can be plotted as a function of ϕ_H, using equation (11) and the assumed values for θ and ω. Next the necessary corresponding force to weight ratios F_A/W and F_H/W can be plotted using (7) and (8). Also the resultant projected drag angles ε_{IA} and ε_{IH} and the required lift elevation angles ϕ_{LA} and ϕ_{LH} can be plotted using (3), (4), (5) and (6), and the assumed drag angles ε_A and ε_H. Following this, the apparent wind angle $\beta = \varepsilon_{IA} + \varepsilon_{IH}$ can be plotted. Finally the expressions dependent on β in Table 1 for relative speeds on various courses can be evaluated. In Figure 9 the relative speeds $V_{S_{\max}}/V_T$ and $V_{mg_{\max}}/V_T$, and their corresponding course angles, have been plotted.

Now suppose the example craft has a gross weight of 175 kg, so W=175 kgf, and is operating in true wind V_T=10 kn. If the craft could sail with apparent wind β =20° then from Figures 2 and 9 it can be seen that a maximum speed $V_{S_{\max}}$ ≈30 kn would be possible, and the best speed made good upwind would be $V_{mg_{\max}}$ ≈10 kn on course γ≈55°. The elevations ϕ_A≈50° and ϕ_H≈15° are required to achieve β=20°. From Figure 9 it can be seen that this would require F_A slightly greater than W=175 kgf. On course γ≈55°, with β=20°, the apparent wind would be V_A≈25 kn. Suppose the craft has an aerofoil area S_A =15 m^2. Assuming a coefficient of lift C_L=1.2 and applying (10) gives sufficient lift L_A ≈180 kgf.

In winds too light for the craft to become airborne, there are niches in which existing craft may have an advantage. For displacement mode sailing, longer craft have a Froude number advantage. Craft that can deploy massive light air rigs may be able to achieve a force to weight ratio advantage.

7 Conclusion

It has been shown that the system of forces acting on any sail craft at equilibrium can be reduced to an equivalent system of three forces representing the net aerodynamic, hydro and gravitational forces. The resultant force lines of action lie in a vertical plane and are concurrent. These are sufficient conditions for equilibrium, it is not necessary to give separate consideration to heeling, pitching and righting moments. There is a direct geometrical relationship between the resultant forces and the velocity triangle. This relationship leads to the fundamental equation (9), which seems to be new, relating the apparent wind angle β to the drag angles ε_A and ε_H and the force elevation angles ϕ_A and ϕ_H. This single equation encapsulates the factors controlling performance, namely the aerodynamic and hydro lift to drag and force to weight ratios. Note that equation (2) relates the apparent wind angle β to the projected drag angles, not the drag angles themselves.

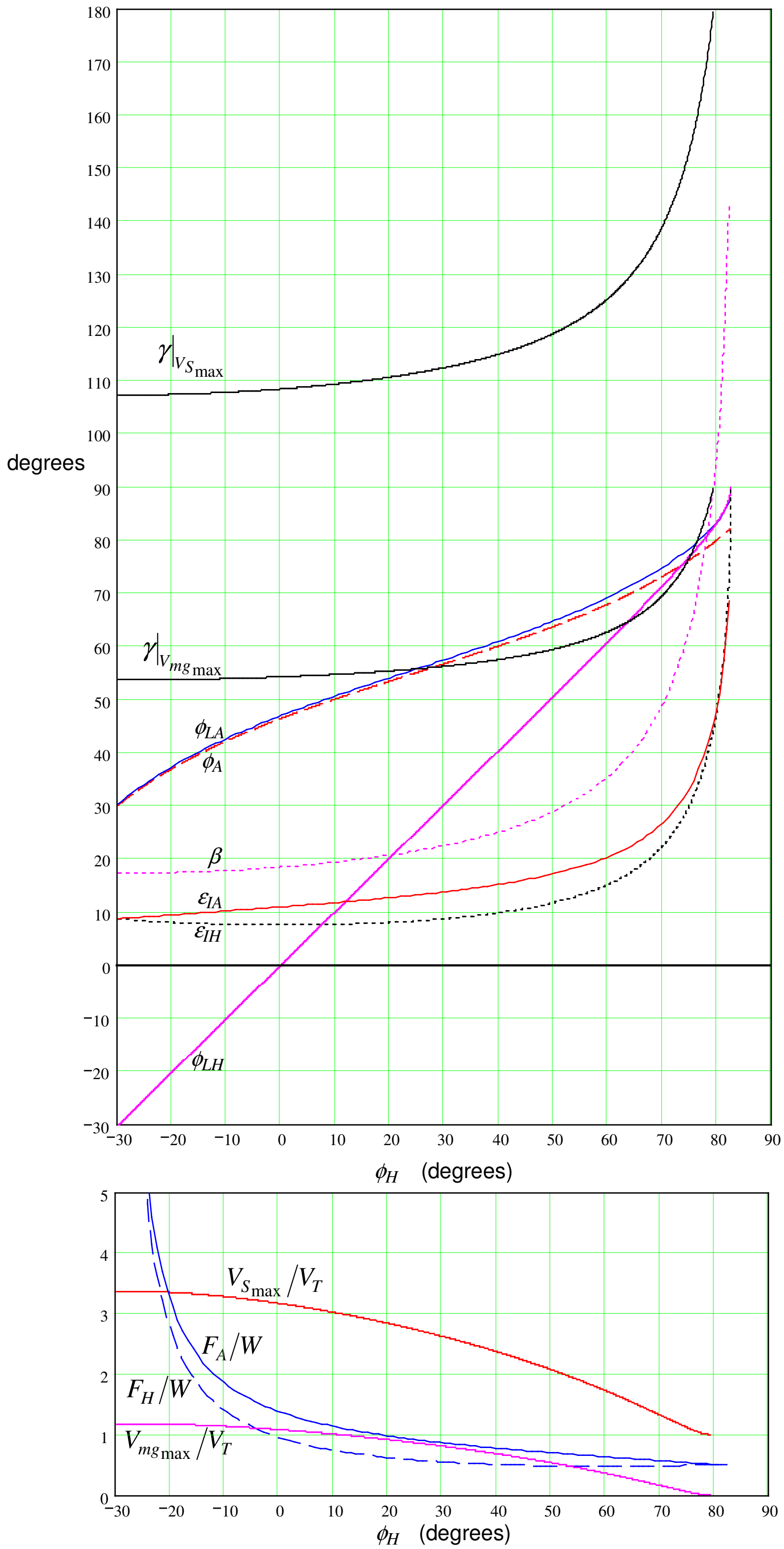

Figure 9 *Angles, velocities, forces as functions of* ϕ_H .

Equation (1), which seems to be new, indicates the extremes to be encountered in the V_A^2/V_S^2 ratio as course γ changes. Sail craft must be able to accommodate these extremes. A benefit of decreasing apparent wind angle β is a corresponding reduction in the variation of this ratio.

The achievement of high performance and stable equilibrium imposes certain general design requirements on the relative locations of the centre of gravity *CG*, the aerodynamic centre of pressure CP_A and the hydro centre of pressure CP_H, and the elevations of the aerodynamic lift ϕ_{LA} and the hydro lift ϕ_{LH}.

A hydrofoil sail craft has been designed with minimal necessary complexity that generally accommodates all of the attributes required for high performance and stability of equilibrium. The craft is designed to operate on both tacks in unsheltered waters over a wide range of true wind speed V_T and course γ. The craft should have superior performance to all existing types of high performance sail craft on all courses in most conditions. The feasibility is demonstrated by example calculations showing exceptional performance.

References

1. Beer, F P and Johnston, E R, *Vector Mechanics for Engineers – Statics*, McGraw-Hill, Fourth Edition 1984.

2. Thompson, Sir William (Lord Kelvin) and Tait, P G, *Principles of Mechanics and Dynamics*, first published as *Treatise on Natural Philosophy* by Cambridge University Press in 1879, last revised 1912, Dover Publications unabridged and unaltered edition 1962.

3. Bethwaite, F, *High Performance Sailing*, International Marine, 1993.

4. Larsson, L and Eliasson, R E, *Principles of Yacht Design*, Adlard Coles Nautical, London, Second Edition, 2000.

5. Lanchester, F W, *Aerodynamics*, Vol 1 p 431, A Constable and Co, London, 1907.

6. Marchaj, C A, *Aero-hydrodynamics of sailing*, Adlard Coles Nautical, London, Second Edition 1988, reprinted 1993.

7. Barkla, H M, *Physics of sailing* (*Encyclopaedic Dictionary of Physics - Supplementary Vol 4*) Pergamon Press, 1971.

8. Perry, J, *AYRS Newsletter*, p 7, October 1998.

9. Bourn, *Hydrofoil Sail Craft*, PCT international patent application, priority date 2 November 1998, publication number WO 00/26083, 11 May 2000.